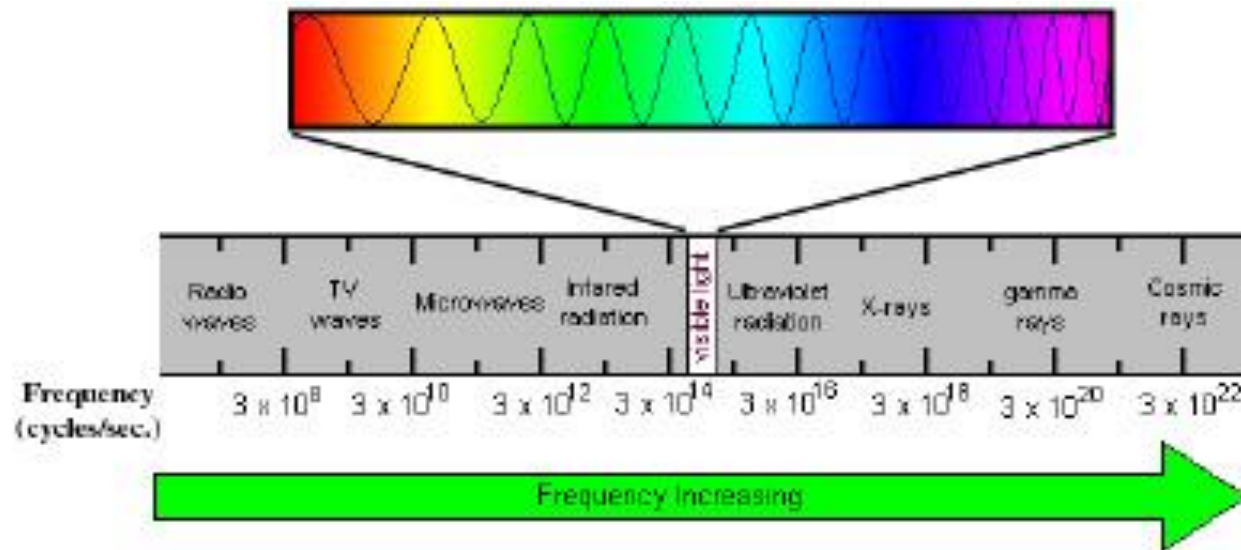


Digital television

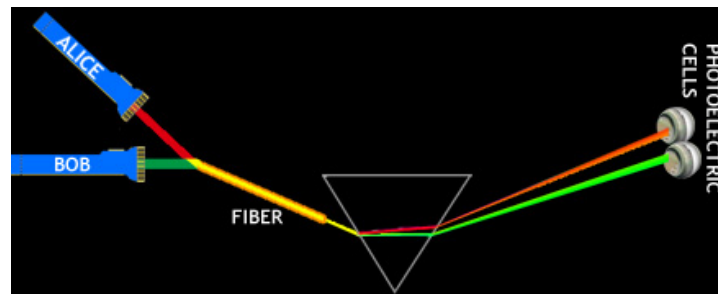
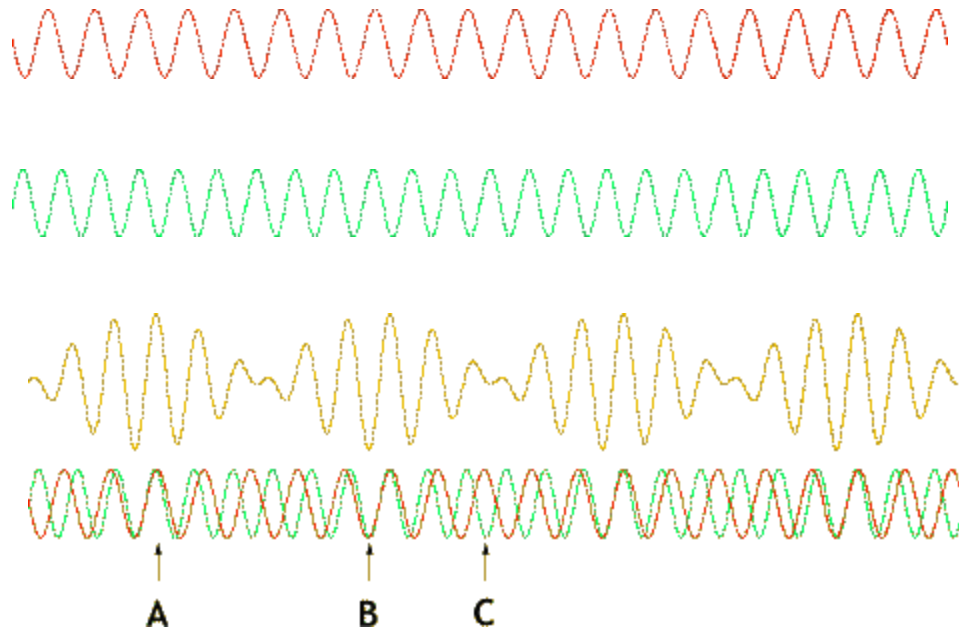
Modulation techniques

- Electromagnetic waves
 - Analog modulation
 - Amplitude modulation
 - Angle modulation
 - Frequency modulation
 - Phase modulation
 - Digital modulation
 - On-Off keying
 - Amplitude shift keying
 - Phase shift keying
 - Quadrature amplitude modulation
-

Electromagnetic waves



Electromagnetic waves



Modulation

"modulate" is "To adjust or adapt to a certain proportion."

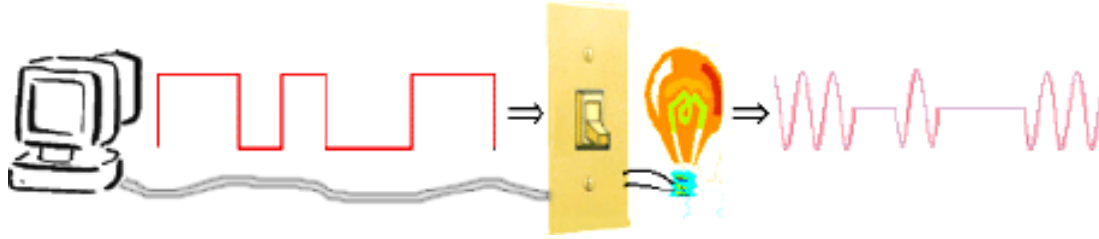
"Linear" modulation

$$x_c(t) = A_c (1 + \mu x(t)) \cos \omega_c t$$

Exponential modulation

$$\begin{aligned} x_c(t) &= A_c \cos(\omega_c t + \phi(t)) \\ &= A_c \cos \Theta_c(t) = A_c \operatorname{Re} [e^{j\Theta_c(t)}] \end{aligned}$$

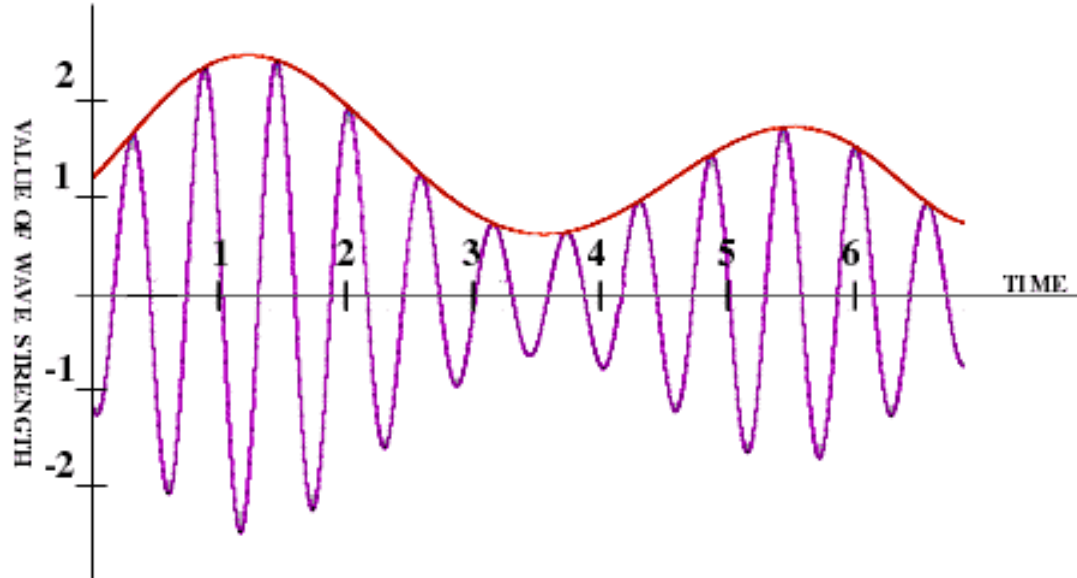
Amplitude modulation



E.g..

- AM radio stations
- Analog television

Amplitude modulation



$$x_c(t) = A_c m(t) \cos 2\pi f_c t \quad (3.1)$$

$$X_c(f) = \frac{A_c}{2} [M(f + f_c) + M(f - f_c)] \quad (3.2)$$

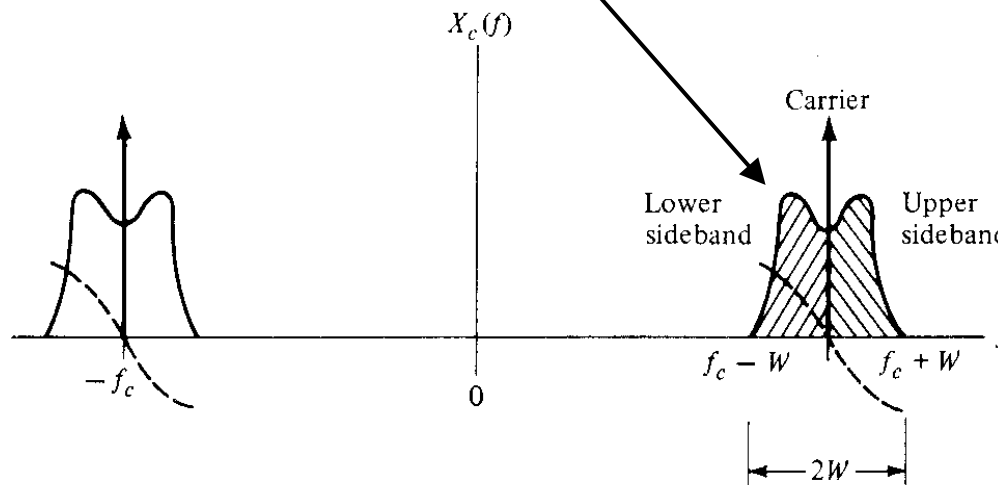
AM: waveforms and bandwidth

- AM in frequency domain:

$$\begin{aligned}
 x_c(t) &= A_c [1 + \mu x_m(t)] \cos(\omega_c t) \\
 &= \underbrace{A_c \cos(\omega_c t)}_{\text{Carrier}} + \underbrace{\mu x_m(t) \cos(\omega_c t)}_{\text{Information carrying part}}
 \end{aligned}$$

$$X_c(f) = \underbrace{A_c \delta(f - f_c) / 2}_{\text{Carrier}} + \underbrace{\mu A_c X_m(f - f_c) / 2}_{\text{Information carrying part}} \quad f > 0 \text{ (for brief notations)}$$

- AM bandwidth is twice the message bandwidth W :



Phase modulation (PM)

Carrier Wave (CW) signal: $x_c(t) = A_c \cos(\underbrace{\omega_c t + \phi(t)}_{\theta_c(t)})$

- In exponential modulation the modulation is “in the exponent” or “in the angle”

$$x_c(t) = A_c \cos(\theta_c(t)) = A_c \operatorname{Re}[\exp(j\theta_c(t))]$$

- Note that in exponential modulation superposition does not apply:

$$x_c(t) = A \cos\{\omega_c t + k_f [a_1(t) + a_2(t)]\}$$

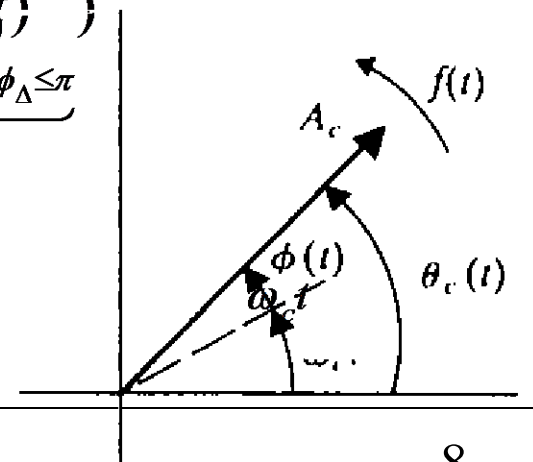
$$\neq A \cos \omega_c t + A \cos k_f [a_1(t) + a_2(t)]$$

- In **phase modulation** (PM) carrier phase is linearly proportional to the modulation amplitude:

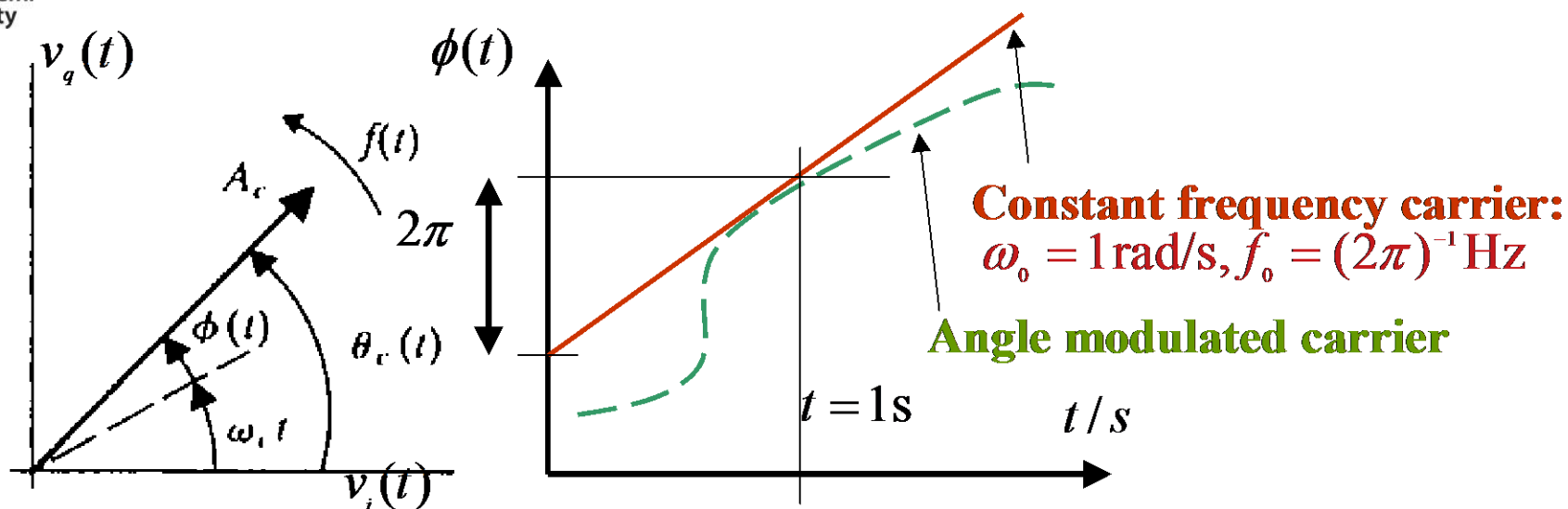
$$x_{PM}(t) = A_c \cos(\omega_c t + \underbrace{\phi(t)}_{\phi_{\Delta} x(t), \phi_{\Delta} \leq \pi})$$

- Angular phasor has the instantaneous frequency (**phasor rate**) $\theta_c(t)$

$$\omega = 2\pi f(t)$$



Instantaneous frequency



- Angular frequency ω (rate) is the derivative of the phase (the same way as the velocity $v(t)$ is the derivative of distance $s(t)$)
- For continuously changing frequency instantaneous frequency is defined by differential changes:

$$\omega(t) = \frac{d\phi(t)}{dt} \quad \phi(t) = \int_{-\infty}^t \omega(\alpha) d\alpha$$

Compare to linear motion:

$$v(t) = \frac{ds(t)}{dt} \left(\approx \frac{s_2(t) - s_1(t)}{t_2 - t_1} \right)$$

Frequency modulation (FM)

- In frequency modulation carrier **instantaneous frequency** is linearly proportional to modulation frequency:

$$\begin{aligned}\omega &= 2\pi f(t) = d\theta_c(t) / dt \\ &= 2\pi[f_c + f_\Delta x(t)]\end{aligned}$$

- Hence the FM waveform can be written as

$$x_c(t) = A_c \cos(\underbrace{\omega_c t + 2\pi f_\Delta \int_{t_0}^t x(\lambda) d\lambda}_{\theta_c(t)}, t \geq t_0 \quad \phi(t) = \int_{-\infty}^t \omega(\alpha) d\alpha$$

← integrate

- Note that for FM

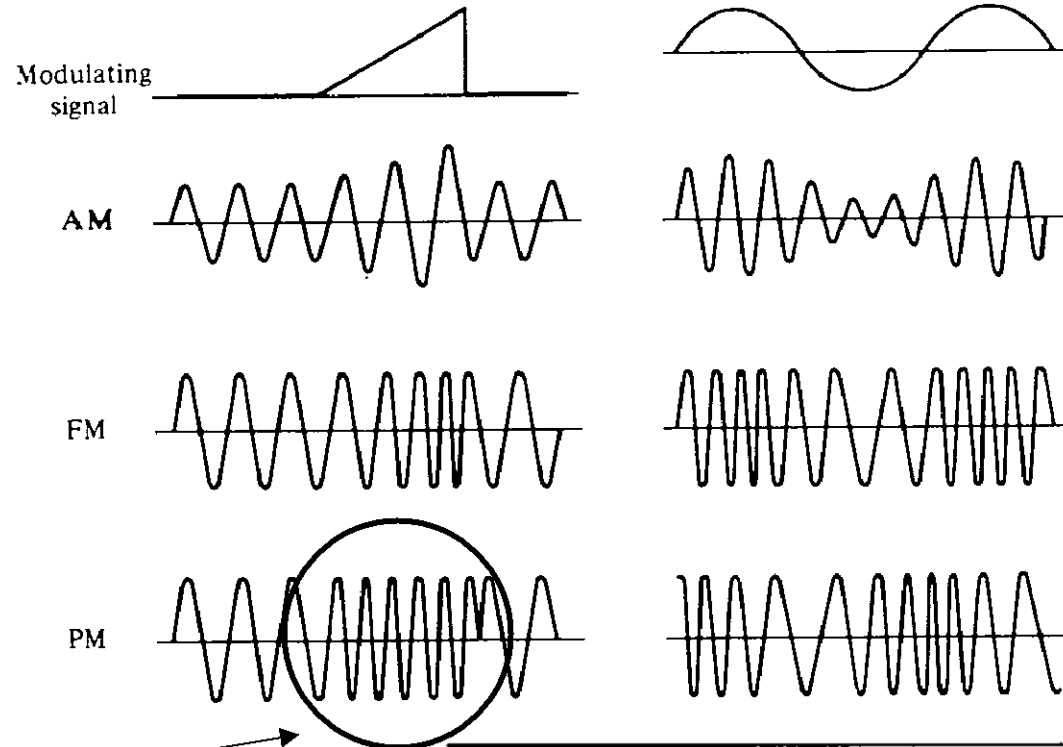
$$f(t) = f_c + f_\Delta x(t)$$

and for PM

$$\phi(t) = \phi_\Delta x(t)$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi} \phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int_{t_0}^t x(\lambda) d\lambda$	$f_c + f_\Delta x(t)$

AM, FM and PM waveforms



Constant frequency follows
constant modulation waveform
derivative

$$x_{PM}(t) = A_c \cos(\omega_c t + \phi_\Delta x(t))$$

$$x_{FM}(t) = A_c \cos(\omega_c t + 2\pi f_\Delta \int_t x(\lambda) d\lambda)$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi} \phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int_t x(\lambda) d\lambda$	$f_c + f_\Delta x(t)$

FM Bandwidth

Normally calculated using Carlsons rule

$$B = 2 (D + 1)W$$

where W is the maximum modulation frequency and
 D is the deviation ratio $D = f_{\Delta} / W$

f_{Δ} is the peak frequency deviation

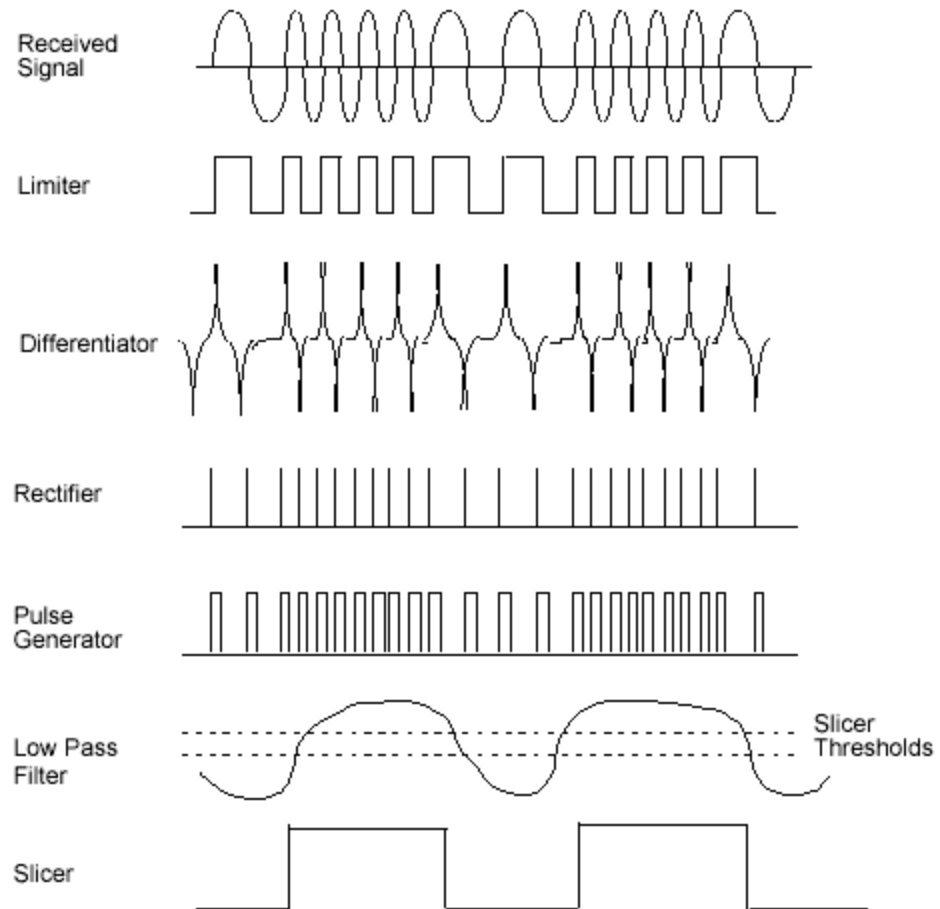
FM Radio: $f_{\Delta} = 75$ kHz, $W = 15$ kHz

$$B = 2 * (75 / 15 + 1) * 15 \text{ kHz} = 180 \text{ kHz}$$

FM demodulation - Example

Zero-crossing
based demodulation

Other:
PLL



Comparison of carrier wave modulation systems

Type	$b = B_T/W$	$(S/N)_D/\gamma$	γ_{th}	DC	Complexity	Comments	Typical applications
Baseband	1	1	...	No†	Minor	No modulation	Short-haul links
AM	2	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$	20	No	Minor	Envelope detection $\mu \leq 1$	Broadcast radio
DSB	2	1	...	Yes	Major	Synchronous detection	Analog data, multiplexing
SSB	1	1	...	No	Moderate	Synchronous detection	Point-to-point voice, multiplexing
VSF	1+	1	...	Yes	Major	Synchronous detection	Digital data
VSF + C	1+	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$	20	Yes‡	Moderate	Envelope detection $\mu < 1$	Television video
PM§	$2M(\phi_\Delta)$	$\phi_\Delta^2 S_x$	$10b$	Yes	Moderate	Phase detection $\phi_\Delta \leq \pi$	Digital data
FM§¶	$2M(D)$	$3D^2 S_x$	$10b$	Yes	Moderate	Frequency detection	Broadcast radio, microwave relay, satellite systems

† Unless direct-coupled.

‡ With electronic DC restoration.

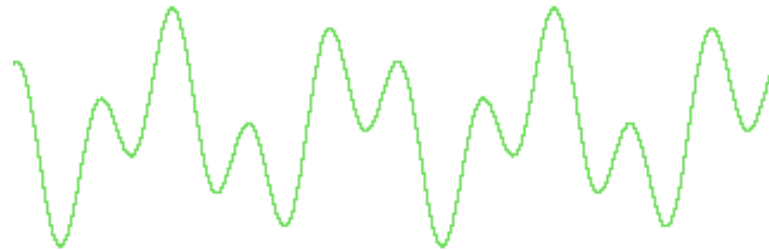
§ $b \geq 2$.

¶ Deemphasis not included.

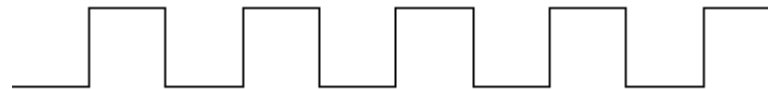
Digital modulation

On-off keying (Binary Amplitude Key Shifting) bandwidth?

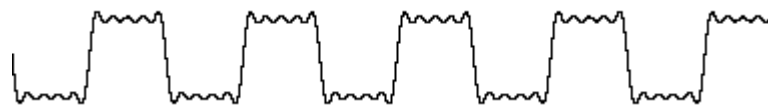
2 analog signals



digital signal (010101...)



digital signal approximated
using 5 sinus waveforms



$$\left[\sin f_m x + \frac{\sin 3\pi f_m x}{3} + \frac{\sin 5\pi f_m x}{5} + \frac{\sin 7\pi f_m x}{7} + \frac{\sin 9\pi f_m x}{9} \right] \text{ e.g. keeping 5 components } \rightarrow B = 18 f_m$$

Shannon's theorem

The capacity C of a channel is

$$C = B \log_2 \left(1 + \frac{S}{N} \right).$$

where B is the bandwidth and S/N is the signal to noise ratio (given in watts/watts)

The dB scale:

$$\text{SNR}_{\text{dB}} = 10 \log_{10} (\text{SNR}) \text{ (dB)}$$

PAL analog 8MHz studio eq. $S/N = 65 \text{ dB} \rightarrow 149 \text{ Mbit/s}$

PAL analog 8MHz broadcast $S/N = 21 \text{ dB} \rightarrow 59 \text{ Mbit/s}$

Practical for digital applications $15 \text{ dB} \rightarrow 30 \text{ Mbit/s}$

Normal digital applications: $6 \text{ bps / Hz} \rightarrow$ today often 8 bps/Hz

Short notes on dB

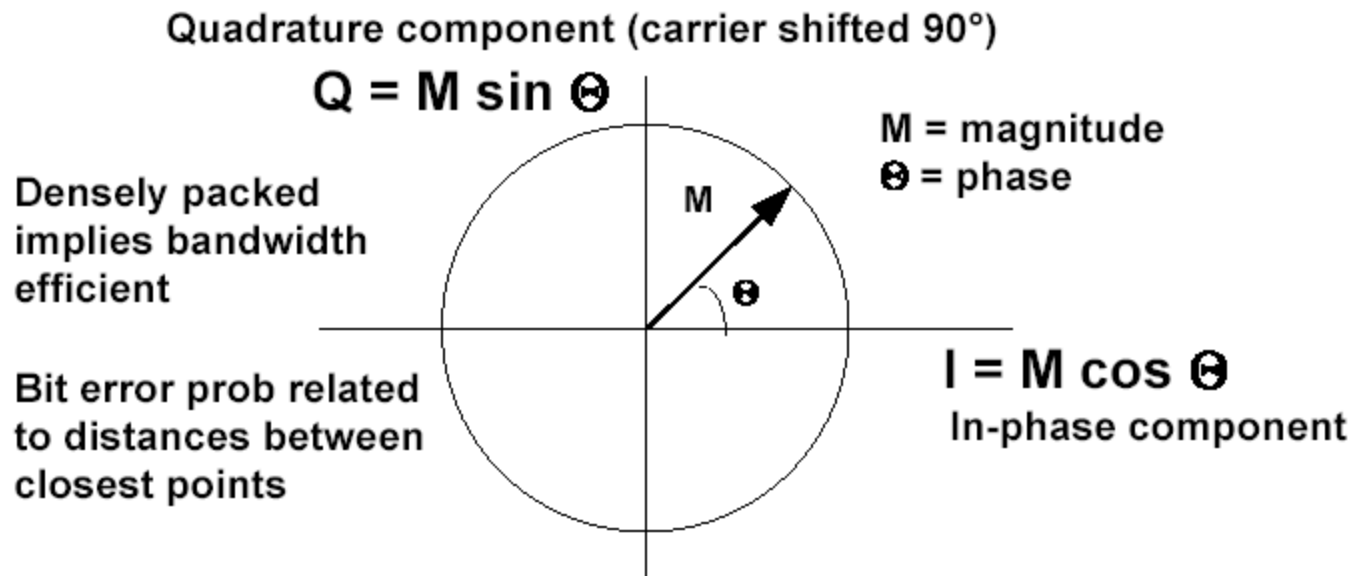
- Generally: magnitude of a physical quantity (usually power or intensity) relative to a certain reference value
- i.e. signal (S) to noise (N) – S/N – given in dB form
$$SNR_{dB} = 10 \log_{10} (S/N) \text{ (dB)}$$

e.g. $SNR_{dB} = 0 \text{ dB} \rightarrow S/N = 1$

$$SNR_{dB} = 2 \text{ dB} \rightarrow S/N = 10^{(2/10)} = 1,58$$
- Voltage
 - dBv – voltage relative to 1 V
 - dBmV – voltage relative to 1 mV
- Radio power
 - dBm – (also dBmW) power ratio in decibels (dB) of measured power referenced to one milliwatt (mW)

Digital modulation

- Modify carriers amplitude and/or phase (and frequency)
- Constellation: Vector notation / polar coordinates

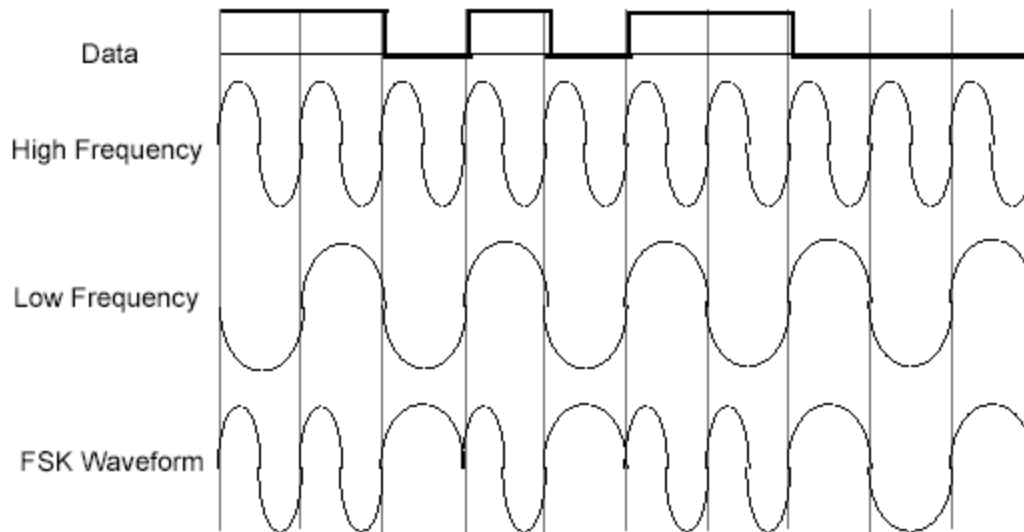
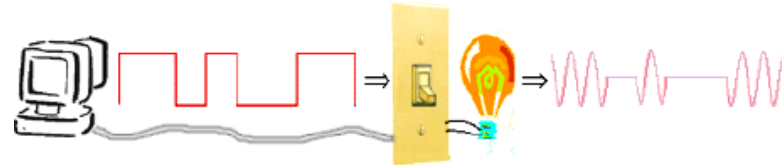


Modulation scheme - considerations

- High spectral efficiency
- High power efficiency
- Robust to multipath effects
- Low cost and ease of implementation
- Low carrier-to-cochannel interference ratio
- Low out of band radiation
- Constant or near constant envelope
 - Constant: Only phase is modulated
 - Non-constant: phase and amplitude is modulated

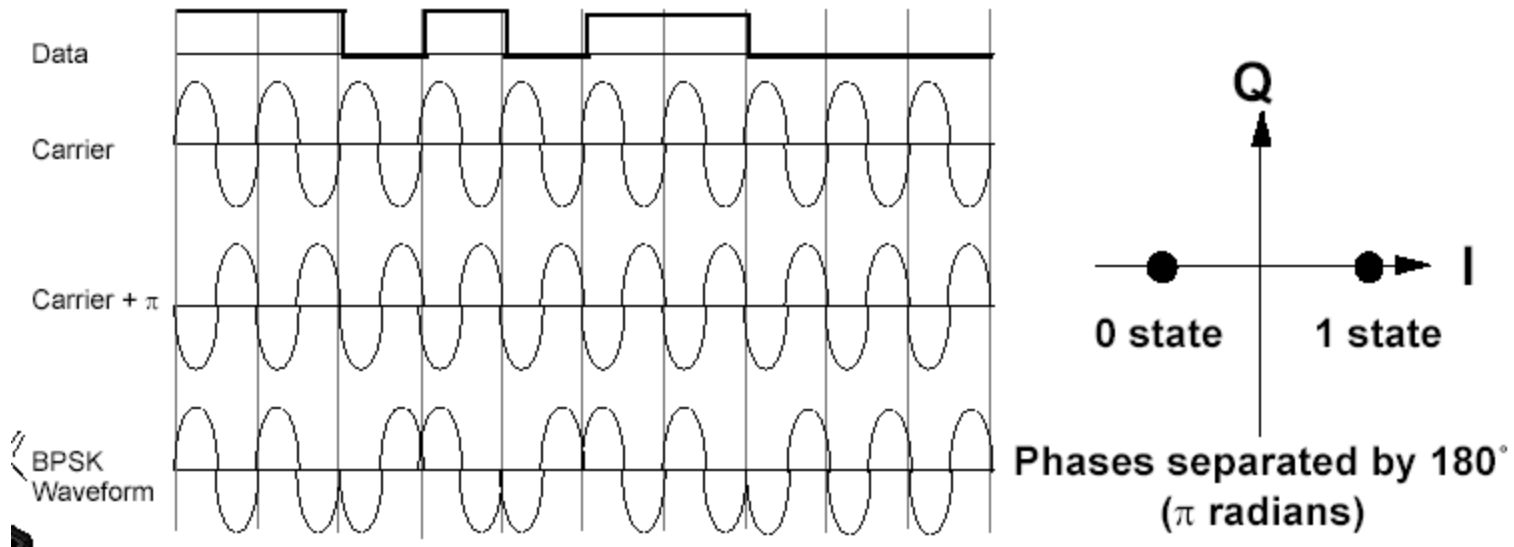
Binary modulations

- Amplitude shift keying (ASK)
Transmission on-off
- Frequency shift keying (FSK)



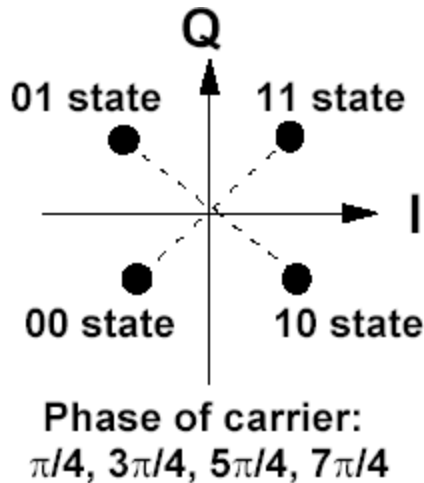
Binary modulations

- Binary phase shift keying (BPSK)
 - Simple to implement, inefficient use of bandwidth
 - Very robust, used in satellite communications



Phase key shifting

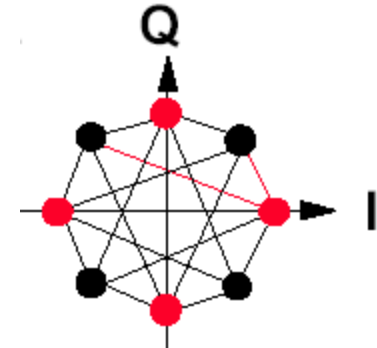
- Quadrature Phase Shift Keying (QPSK)
 - Multilevel modulation technique: 2 bits per symbol
 - More spectral efficiency, more complex receiver



Output waveform is
sum of modulated \pm
Cosine and \pm Sine wave

$\pi / 4$ – Shifted QPSK

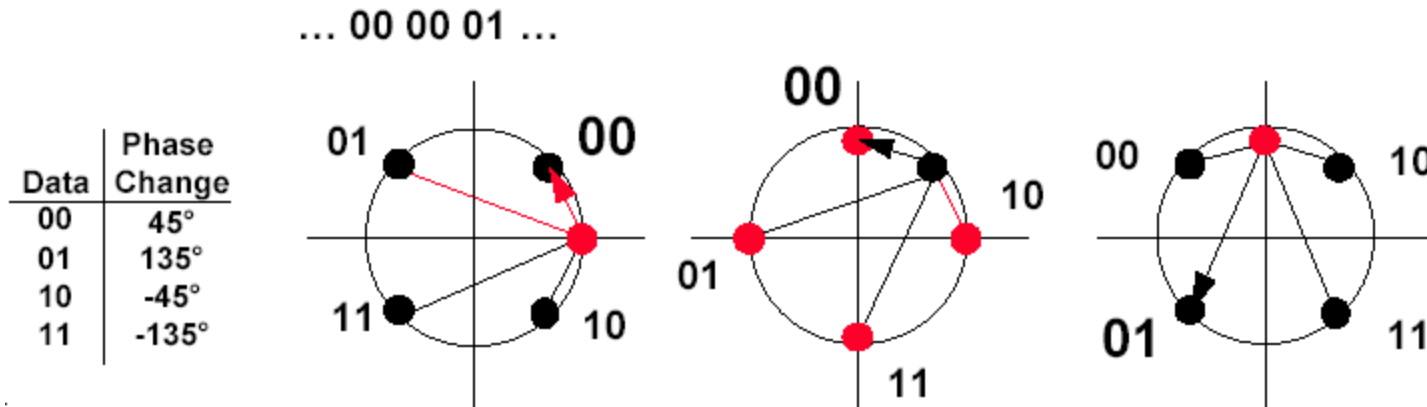
- Variation of QPSK
 - Restricted carrier phase transitions to $\pm \pi/4$ and $\pm 3\pi/4$
 - Signalling elements selected in turn from two QPSK constellations each shifted by $\pi/4$
- Popular in Second Generation Systems
 - North American Digital Cellular (1.62 bps / Hz)
 - Japanese Digital Cellular System (1.68 bps / Hz)



$\pi / 4$ – Shifted QPSK

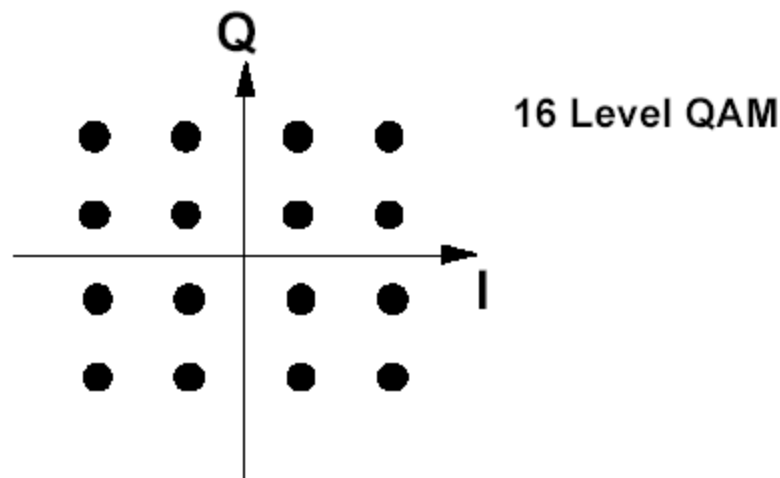
• Advantages

- Two bits per symbol
- Phase transitions avoid center of diagram, remove some design constraints on receiver
- Always a phase change between symbols, leading to self-clocking



Quadrature Amplitude Modulation

- Quadrature Amplitude Modulation (QAM)
 - Amplitude modulation on both quadrature carriers
 - 2^n discrete levels, if $n=2$ -> same as QPSK
- Extensively used in microwave links
- DVB-T uses QAM



Quadrature Amplitude Modulation

4 bits / symbol

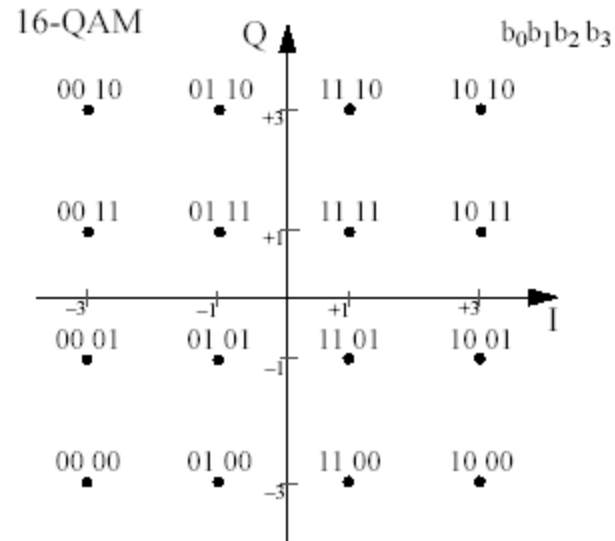


Table 84—16-QAM encoding table

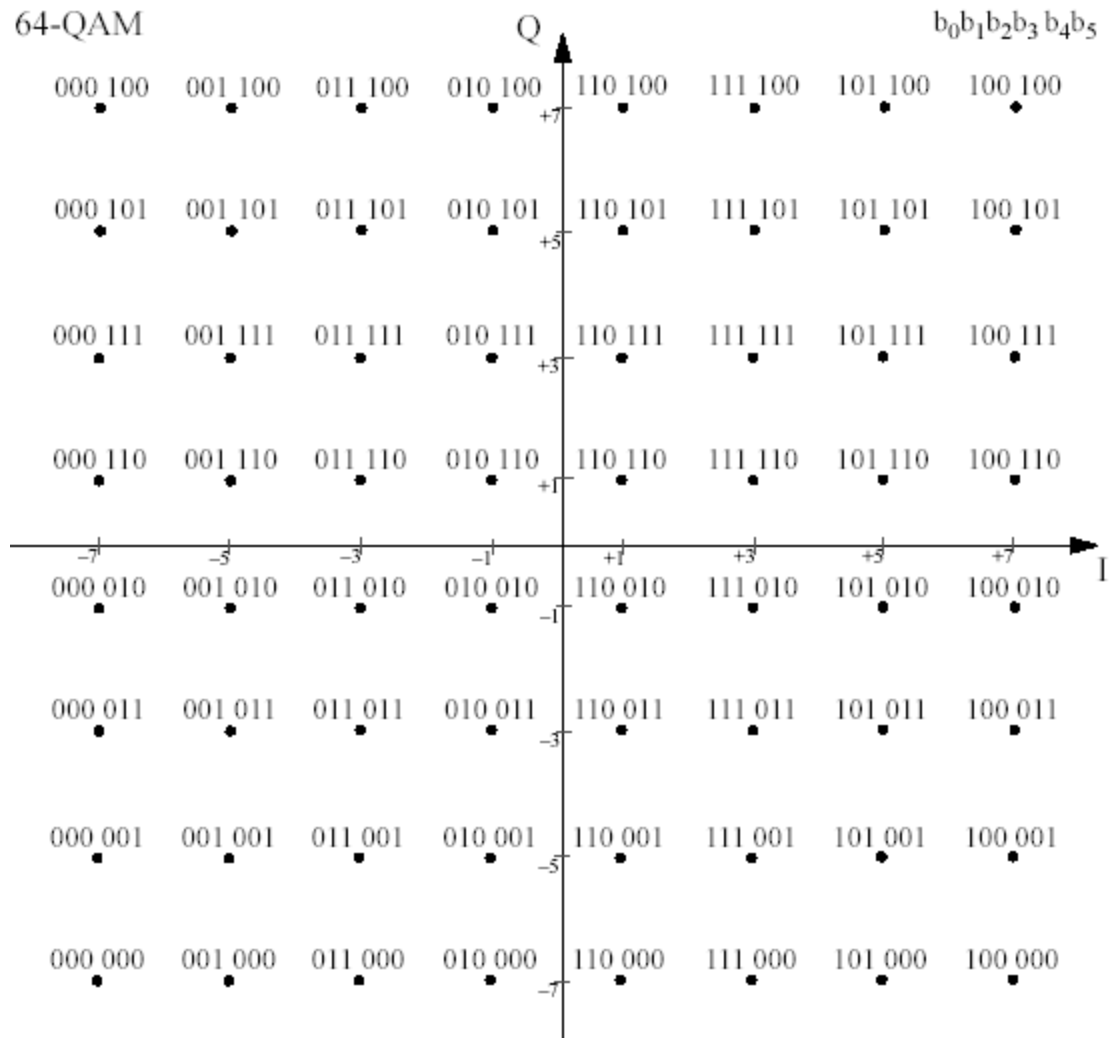
Input bits ($b_0 b_1$)	I-out	Input bits ($b_2 b_3$)	Q-out
00	-3	00	-3
01	-1	01	-1
11	1	11	1
10	3	10	3

Quadrature Amplitude Modulation

6 bits / symbol

Table 85—64-QAM encoding table

Input bits ($b_0 b_1 b_2$)	I-out	Input bits ($b_3 b_4 b_5$)	Q-out
000	-7	000	-7
001	-5	001	-5
011	-3	011	-3
010	-1	010	-1
110	1	110	1
111	3	111	3
101	5	101	5
100	7	100	7



Gray coding

- Present integers, represented in binary format, in such an order, that adjacent integers differ only in one position
- In QAM modulation, complex numbers are usually represented by their in-phase (I) and quadrature (Q) components
- Now, hamming distance represent physical distance between constellation points

